

U.S. Patent Application
Docket #34650-00448USPT
Ericsson Ref. P11573US1

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DETERMINING CORRELATIONS OF RECEIVED SEQUENCES TO MULTIPLE
KNOWN SEQUENCES IN A COMMUNICATIONS SYSTEM

BACKGROUND OF THE INVENTION

Technical Field of the Invention

The present invention relates in general to the field
of communications, and in particular, by way of example but
not limitation, to determining correlations between received
sequences and multiple training sequences in an efficient
manner that minimizes the number of required mathematical
operations.

Description of Related Art

Many people and organizations increasingly rely on wireless communication for safety, convenience, and productivity, as well as simple conversational pleasure. One example of wireless communication is cellular communication. Cellular phone use has proliferated as the size of mobile terminals (MTs) and the cost for service subscriptions as well as air time have decreased. As cellular phone use has proliferated and the allocated radio frequency (RF) spectrum has become correspondingly more crowded, it has become ever more important to efficiently utilize the available RF spectrum.

Cellular communication systems, such as the Global System for Mobile Communications (GSM), therefore need to efficiently reuse the RF spectrum in order to attempt to maximize the capacity of the system. Consequently, such systems are often Carrier-to-Interference (C/I) limited; in other words, co-channel interferers are often the factor limiting capacity in a system. In conventional systems, co-

channel interferers are addressed in the demodulation process as unknown white noise.

However, it has been observed that better receiver performance (e.g., a lower bit error rate (BER)) can be obtained by using knowledge of the interfering signal in the data recovery process. In fact, studies have indicated that receiver performance in C/I-limited cellular systems can be significantly improved by using co-channel interference rejection (IR) techniques. Co-channel IR techniques require an initial identification of the interferers, which may be accomplished by finding the training sequences of all received interfering signals. The training sequences may be found by correlating each received sequence with all possible training sequences that are used in the C/I-limited cellular system.

Correlation of a known (e.g., training) sequence with a sequence of samples can be computed according to the correlation definition; in other words, the correlation may be computed as an inner product of the known sequence with the sample sequence at each of a number of offsets.

Unfortunately, the computational complexity of the correlation calculation grows with the training sequence length, with the number of training sequences, and with the number of offsets considered. It should be noted that a
5 large number of offsets must ordinarily be considered because different base stations (BSs) will not generally be synchronized. The resulting large number of operations required for solving the correlation equation causes a high cost in terms of hardware requirements, processing time
10 delays, and power dissipation demands.

SUMMARY OF THE INVENTION

5 The deficiencies of the prior art are overcome by the method, system, and apparatus of the present invention. For example, as heretofore unrecognized, it would be beneficial to reduce the number of operations required when calculating a correlation between a known sequence and a sample sequence. In fact, it would be beneficial if the number of add/subtract operations per additional input sample required to calculate correlates were reduced by eighty percent (80%) when
10 identifying potential interferers in a wireless communications system.

In accordance with an embodiment of the present invention, the number of operations for calculating correlates may be significantly reduced by taking advantage
15 of certain properties of the training sequences of a given wireless communications system. In a GSM wireless communications system, for example, a straightforward implementation requires two hundred (200) add/subtract-operations per additional input sample to calculate the

correlates with eight (8) sequences, each of which is
composed of twenty-six (26) elements. In contrast, an
implementation in accordance with an embodiment of the
present invention advantageously only requires forty (40)
5 add/subtract-operations per additional input sample.

Interference rejection improves the extraction of
information from a desired signal that is received along with
interfering signals. For an interfering signal to be
rejected, the interfering signal must first be identified.
10 An interfering signal may be identified based on a comparison
between a received sequence of signals and expected training
sequences. Specifically, if a correlation between a received
sequence of signals, or an offset thereof, and one of
multiple training sequences is sufficiently high, then an
15 interfering signal has been identified, and it may therefore
be subsequently rejected. A correlation between a received
sequence of signals and a training sequence may be determined
based on the definition (e.g., a summation of the products
of values of the received sequence and values of the training
20 sequence). A determination of the correlation equation, if

performed in a straightforward manner based on the definition, is a processing intensive task as a result of the number of mathematical operations involved. The present invention enables the number of mathematical operations to
5 be significantly reduced.

In a first embodiment in accordance with the present invention, the correlation equation is manipulated to produce a sum that may be reused for each training sequence. In a second embodiment, the number of mathematical operations
10 is reduced by manipulating the correlation equation such that many of the products would be zero if calculated. In a third embodiment, repeated subsequences in the training sequences are relied on to reduce the number of products that must be calculated. In a fourth embodiment, common subexpressions
15 are eliminated. In a fifth embodiment, the negative of the correlation equation is selectively calculated when the corresponding number of mathematical operations is less than when calculating the standard, non-negative correlation. In a sixth embodiment to reduce the number of mathematical
20 operations involved in calculating the correlation result,

any two or more of the first through the fourth embodiments
are synergistically combined so as to further reduce the
number of mathematical operations involved in determining the
correlates. The results of one or more of these
5 manipulations may be advantageously implemented in a receiver
to improve the calculation of correlates, which consequently
reduces the processing intensity involved in identifying
potential interferers.

The technical advantages of the present invention
10 include, but are not limited to, the following. It should
be understood that particular embodiments may not involve
any, much less all, of the following exemplary technical
advantages.

An important technical advantage of the present
15 invention is that it reduces the hardware requirements,
processing time delays, and power dissipation demands
otherwise involved when identifying potential interferers.

Another important technical advantage of the present
invention is that it enables the calculation of the
20 correlates of a signal using up to eighty percent (80%) fewer

add/subtract operations per additional input sample in a GSM
implementation.

Yet another important technical advantage of the present
invention is the ability to tailor specific embodiments to
5 the training sequences of other communication standards.

The above-described and other features of the present
invention are explained in detail hereinafter with reference
to the illustrative examples shown in the accompanying
drawings. Those skilled in the art will appreciate that the
10 described embodiments are provided for purposes of
illustration and understanding and that numerous equivalent
embodiments are contemplated herein.

BRIEF DESCRIPTION OF THE DRAWINGS

A more complete understanding of the method, system, and apparatus of the present invention may be had by reference to the following detailed description when taken in
5 conjunction with the accompanying drawings wherein:

FIGURE 1 illustrates a portion of an exemplary wireless communications system with which the present invention may be advantageously employed;

FIGURE 2 illustrates an exemplary base station in
10 accordance with the present invention;

FIGURE 3A illustrates a calculation of a correlate in graphical form according to the definition of a correlate;

FIGURE 3B illustrates a table of training sequences in accordance with both the current GSM standard and an
15 embodiment of the present invention;

FIGURE 3C illustrates an exemplary calculation of a correlate in graphical form in accordance with first and second embodiments of the present invention;

FIGURE 3D illustrates another exemplary calculation of a correlate in graphical form in accordance with the first and second embodiments of the present invention;

FIGURE 3E illustrates an exemplary calculation of a
5 correlate in graphical form in accordance with a third embodiment of the present invention;

FIGURE 4 illustrates an exemplary method in flowchart form for rejecting interfering signals in accordance with the present invention;

10 FIGURE 4A illustrates an exemplary correlation equation manipulation in accordance with the first embodiment of the present invention;

FIGURE 4B illustrates an exemplary correlation equation manipulation in accordance with the second embodiment of the
15 present invention;

FIGURE 4C illustrates an exemplary correlation equation manipulation in accordance with the third embodiment of the present invention;

FIGURE 4D illustrates an exemplary correlation equation manipulation in accordance with a fourth embodiment of the present invention;

FIGURE 4E illustrates an exemplary correlation equation
5 manipulation in accordance with a fifth embodiment of the present invention; and

FIGURE 5 illustrates a table of an exemplary correspondence between first and second-order terms and correlates in accordance with the present invention.

FIGURE 5

DETAILED DESCRIPTION OF THE DRAWINGS

In the following description, for purposes of explanation and not limitation, specific details are set forth, such as particular circuits, logic modules
5 (implemented in, for example, software, hardware, firmware, some combination thereof, etc.), techniques, etc. in order to provide a thorough understanding of the invention. However, it will be apparent to one of ordinary skill in the art that the present invention may be practiced in other
10 embodiments that depart from these specific details. In other instances, detailed descriptions of well-known methods, devices, logical code (e.g., hardware, software, firmware, etc.), etc. are omitted so as not to obscure the description of the present invention with unnecessary detail.

15 An embodiment of the present invention and its advantages are best understood by referring to FIGURES 1-5 of the drawings, like numerals being used for like and corresponding parts of the various drawings.

Aspects of the air interface for the GSM standard will be used to describe an embodiment of the present invention. However, it should be understood that the principles of the present invention are applicable to other wireless communication standards (or systems), especially those that involve training sequences (or their equivalent).

Referring now to FIGURE 1, a portion of an exemplary wireless communications system with which the present invention may be advantageously employed is illustrated generally at 100. The (portion of) wireless communications system 100 includes a cell 105 that is served by a BS 110. The BS 110 typically includes a base transceiver station (BTS) 110A and optionally includes a BS controller (BSC) 110B. Within the cell 105 are multiple MTs 115A and 115B, each of which may be in communication with the wireless network infrastructure as represented by BS 110. Each MT 115 may be, for example, a hand-held cellular phone (e.g., the MT 115A), a vehicle-mounted MT (e.g., the MT 115B), a data terminal with a wireless link (not specifically shown), etc. While only two MTs 115 are shown in the wireless

communications system 100, many more MTs 115 are usually present within the cell 105. Also, it should be noted that the wireless communications system 100 is usually composed of many such cells 105 and BSs 110.

5 Referring now to FIGURE 2, an exemplary base station in accordance with the present invention is illustrated generally at 110. The BS 110 may receive a desired signal 205 along with multiple interfering signals 210 in a C/I-limited wireless communications system. The desired signal
10 205 typically originates from within the cell 105 while the interfering signals 210 typically originate from other cells (not specifically shown) of the wireless communications system 100. The desired signal 205 and the interfering signals 210 may be received at an antenna (e.g., a single
15 antenna, an antenna array, etc.) 215 of the BS 110. The BS 110 also includes a transceiver portion 220, which may be part of the BTS 110A, the BSC 110B, etc. It should be understood that interference identification in accordance with the present invention may be accomplished in other parts

of the wireless communications system 100 (e.g., a Mobile Services Switching Center (MSC) (not specifically shown)).

5 The signals 205 and 210 may be received at the antenna 215 and routed to a receiver 225 of the transceiver portion 220. The receiver 225 forwards the received signals 205 and 210 to an interference identifier 230. Once the interfering signals 210 have been identified, the interfering signals 210 are separated from the desired signal 205 at an interference rejector 235. Subsequently, the desired signal 205 is subjected to further processing (e.g., at block 240). It should be understood that in practice the desired signal 205 and the multiple interfering signals 210 are jointly detected, and only the data/information from the desired signal 205 is retained. The interference identifier 230 may be used to identify the interfering signals 210 based on detection of received sequences (e.g., by correlation with all possible training sequences). It should be noted that the interference identifier 230, the interference rejector 235, and/or the further processing block 240 need not be part of the transceiver portion 220.

5 The interference identifier 230 may include a processing unit 245 and a memory unit 250. The processing unit 245 may be composed of a digital signal processor (DSP) while the memory unit 250 may therefore include instructions for operating the DSP in accordance with the present invention. Alternatively, the processing unit 245 may be composed of an application specific integrated circuit (ASIC), a general-purpose microprocessor, etc. In another alternative, the memory unit 250 may be a circular buffer, a random access memory (RAM) (e.g., dynamic, static, etc.), a programmable read only memory (PROM) (e.g., an electrically-erasable PROM (EEPROM), etc.), etc. Other alternatives, including combinations of the above, are possible. It should be understood that the principles of the present invention are not limited by the type of processing unit(s) 245, memory unit(s) 250, or general logic module(s) (not specifically shown) involved in realizing an embodiment in accordance with the present invention. It should also be understood that the principles of the present invention are applicable to receivers of communication stations in general, such as, for

example, receivers in a MT-type, a BS-type, etc.
communication station.

In wireless communication systems, intersymbol
interference (ISI), which distorts the transmitted signal,
5 is caused by multipath in bandlimited time dispersive
channels. Equalization involves processing a received signal
so as to minimize ISI. Equalizers are periodically updated
to minimize the ISI by evaluating a received sequence as
compared to an expected sequence. Tap weights of the
10 equalizer are adjusted such that the received sequence is
interpreted as the expected sequence to thereby reduce the
ISI for portions of the signal that contain no expected value
(e.g., sequence). The expected sequence is commonly termed
a training sequence. By detecting a training sequence, an
15 adaptive algorithm in a receiver of the wireless
communications system is able to calibrate the receiver until
a subsequent training sequence is received, at which point
the calibration is repeated.

Referring now to FIGURE 3A, a calculation of a correlate
20 in graphical form according to the definition of a correlate

is illustrated generally at 300. The correlate, C_j , of a sequence of received samples, a_n , with respect to a certain training sequence, c_i , $i = 1, \dots, N$, at a certain offset, j , can be defined as follows:

5
$$C_j = \sum_{i=1}^N (a_{i+j} \cdot c_i) \quad (0).$$

FIG. 300

The graphical calculation 300 illustrates how the correlate C_j is calculated from the stream of samples a_n and the training sequence c_i . The stream of samples a_n is position-wise multiplied with the fixed training sequence c_i , $i = 1, \dots, N$. The correlate is computed as the sum of N product terms. It should be understood that the values illustrated in the graphical calculation 300 may correspond to memory locations and that the summing element may correspond to a adder.

10

15 For the graphical calculation 300, N multiplications and $N-1$ additions are needed to carry out the calculation (of Equation (0)). The offset, j , is incremented for every new

sample. For each new value of j , the entire calculation must be performed again. Moreover, when correlates with respect to several training sequences are to be calculated in parallel, the number of operations grows as the number of training sequences increases. For GSM systems where $N=26$, the total number of training sequences is eight (8). Consequently, the overall number of operations per additional input sample is 208 [= 26×8] multiplications and 200 [= $(26-1) \times 8$] additions when correlates are calculated according to the definition as presented by Equation (0). Advantageously, application of the principles of the present invention significantly reduces the required number of overall operations.

Referring now to FIGURE 4, an exemplary method in flowchart form for rejecting interfering signals in accordance with the present invention is illustrated generally at 400. Initially, the receiver receives signal(s) (block 410). The interfering signal(s) from among the received signal(s) are identified (block 420), and the interfering signal(s) may be rejected thereafter (block 430).

To identify the interfering signal(s) (at block 420), the received signal(s) are matched to training sequence(s) (e.g., the training sequences of table 320 (of FIGURE 3B)) (block 435). Manipulated correlate equation(s) are formed from the training sequence(s) and the received sequence(s) by applying the training sequence(s) and the received sequence(s) to manipulated (e.g., modified) correlate equation(s) (block 455). The manipulated correlate equation(s) are/have been manipulated in manners in accordance with the present invention as described in further detail hereinbelow to reduce the number of mathematical operations required to evaluate the correlate equation(s). A manipulated correlate equation implies that a determination of a correlate equation has been optimized using one, two, or more of the below-described embodiments.

After forming the manipulated correlate equation(s) to consequently reduce the processing burden of determining result(s) thereof, the result(s) of the manipulated correlate equation(s) are determined (block 465). In other words, by using the manipulated correlate equation(s) in accordance

with embodiments of the present invention, the number of calculations (e.g., additions) to calculate a correlation is reduced as compared to a straight-forward implementation. Signal(s) associated with/corresponding to a high correlate
5 equation result are identified as at least potential interferers (block 475). The offset at which the received sequence(s) is correlated to the training sequence(s) is iterated (block 485), and blocks 455, 465, and 475 may be repeated (not specifically indicated in flowchart 400 by an
10 arrow) for the next sample of the received sequence(s). The identification of interfering signal(s) (at block 420) may continue thereafter with another training sequence (block 495) as indicated by the arrow 497.

Referring now to FIGURE 3B, a table of training
15 sequences in accordance with both the current GSM standard and an embodiment of the present invention is illustrated generally at table 320. In GSM wireless communications systems, eight (8) training sequences are defined by the current standard. Each of the eight (8) training sequences
20 (designated "Seq. #") includes twenty-six (26) entries

(designated "index 'i'"). As illustrated by the table 320, when bits are mapped to IQ coordinates, the GSM training sequences may be written as a series of "1" and "-1". The training sequences as presented in the table 320 have some special properties that enable the number of calculations required to compute the set of correlates to be reduced in accordance with the present invention.

With regard to the table 320, it is noted that each training sequence, c_i , $i = 1, \dots, N$, is composed of only the values "1" and "-1". It is therefore possible to completely avoid performing any multiplication operations by instead adding (when $c_i = "1"$) and subtracting (when $c_i = "-1"$) the appropriate a_n values from a running sum. The total computation is thus entirely reduced to additions and subtractions.

With regard to the table 320, it is further noted that each training sequence, c_i , $i = 1, \dots, N$, is composed of only two distinct values (namely, "1" and "-1"). The

correlate C_j with respect to a certain sequence a_i at a certain offset j can be written as:

$$\begin{aligned} C_j &= \sum_{i=1}^N (c_i \cdot a_{i+j}) = 2 \sum_{i=1}^N \left(\frac{(c_i + 1)}{2} \cdot a_{i+j} \right) - \sum_{i=1}^N a_{i+j} = \\ &= 2 \sum_{i=1}^N (d_i \cdot a_{i+j}) - \sum_{i=1}^N a_{i+j} = 2 \sum_{i=1}^N (d_i \cdot a_{i+j}) - S_j \end{aligned} \quad (1)$$

The sequence d_i is defined as follows:

$$d_i = \begin{cases} 1 & c_i = 1 \\ 0 & c_i = -1 \end{cases} \quad (2)$$

And S_j is defined as follows:

$$S_j = \sum_{i=1}^N a_{i+j} \quad (3)$$

With respect to a first embodiment in accordance with the present invention, it is noted that the index expression for the a term is symmetric in i and j . Consequently, it is "cheap" to update S_j when the index/offset j has been incremented because the update may be accomplished merely by adding a_{j+26} and subtracting a_j . More generally, S_j may be updated using only two add/subtract operations:

$$S_j \leftarrow S_{j-1} - a_j + a_{j+N} .$$

It should also be noted that these operations should be executed with full precision because any roundoff errors will accumulate. Furthermore, as described further hereinbelow with reference to a second embodiment of the present invention and block 455B (of FIGURE 4B), for those values of i where $d_i = 0$, the value of the corresponding term in the sum goes to zero (0). These zero (0) terms may therefore be omitted from the sum without error.

Referring now to FIGURE 3C, an exemplary calculation of a correlate in graphical form in accordance with first and

second embodiments of the present invention is illustrated generally at 340. The graphical calculation 340 illustrates the calculation of a single correlate C_j according to the Equation (1). Terms where $d_i = 0$ are omitted from the sum.

5 It should be noted that the "multiplications by 1" are shown only for clarity; they need not be actually calculated. As indicated by the "delay" memory location, the value of S_j may be retained and re-used when j is incremented. The graphical calculation 340 reduces the number of add/subtract operations
10 required for each new offset because it is so "cheap" to update S_j .

It is also noted that S_j is independent of the training sequence (values) and that it can therefore be shared among all the correlates. Consequently, when several correlates
15 are computed in parallel, S_j need be computed only once for each new value of j and used for all correlates. In other words, when the correlate equation is expressed as in Equation (1), some computational efforts may be amortized over several correlates. Referring now to FIGURE 3D, another

exemplary calculation of a correlate in graphical form in accordance with the first and second embodiments of the present invention is illustrated generally at 360. Three training sequences c_i ($d_{I/II/III}$) are explicitly shown as being
5 used to calculate three correlates $C_{I/II/III,j}$. Other correlates (e.g., five (5) more for GSM) may also be calculated. As indicated by the "delay" box, the running sum S_j may advantageously be used in the calculation of correlates with respect to several training sequences.

10 Referring now to FIGURE 4A, an exemplary correlation equation manipulation in accordance with the first embodiment of the present invention is illustrated at block 455A. As indicated by block 455A, the above described manipulation of the correlation equation modifies the correlate equation to
15 include a sum (e.g., as in Equation (3)) that is dependent on the received sequences while remaining independent of the training sequences.

With respect to the multiple embodiments in accordance with the present invention, it should be understood that the
20 correlate equation(s) manipulation(s) of blocks 455A-E (of

FIGURES 4A-4E) may be effectuated before a receiver receives signals in use. For example, the manipulated equations may be incorporated into a receiver during the design phase (e.g., in an ASIC, when programming DSP code, etc.), may be
5 determined prior to receiver use by the receiver or an associated processing unit (e.g., in a setup phase of a receiver for a new, modified, or otherwise different standard; in a calibration phase of a receiver; when accommodating new training sequences, etc.), etc. In other
10 words, the manipulations may typically be established prior to utilizing the receiver (e.g., of a MT, BS, etc.). Consequently, the manipulations need not be repeated with any frequency (e.g., usually only as a result of a change to the number of training sequences, their length, their contents,
15 etc.).

With respect to the second embodiment in accordance with the present invention, it is noted that a significant portion of the terms in Equation (1) "drop out" of the calculus (e.g., the products are zero) as a result of the manner in
20 which the training sequences are presented in table 320.

Specifically, to produce the eight (8) correlates, eight (8) sums are calculated (one for each correlate), each of which corresponds to the first summation in Equation (1). Each of these eight (8) sums contains n_d non-zero *first-order terms*,
5 where n_d is the number of ones in the sequence d_i corresponding to the respective training sequence (e.g., for an embodiment with training sequences as in table 320 (of FIGURE 3B)), as presented mathematically in the following equation:

$$n_d = \sum d_i \quad (4)$$

10 Calculation of each sum therefore requires at most $n_d - 1$ additions. About half of the values in each sequence in table 320 are one, which is typically the case. Thus, the mathematical operation count is reduced by approximately one-half in accordance with this second embodiment alone as
15 compared to the straightforward implementation of FIGURE 3A. (It should also be noted that a multiplication by "2" has

been introduced, but multiplication by "2" is a cheap operation.) Referring now to FIGURE 4B, an exemplary correlation equation manipulation in accordance with the second embodiment of the present invention is illustrated at block 455B. As indicated by block 455B, the above described manipulation of the correlation equation modifies the correlate equation(s) so that all products corresponding to at least one value of the training sequence(s) become zero.

With respect to a third embodiment in accordance with the present invention, it is noted that the 26-bit GSM training sequences as listed in table 320 are based on 16-bit "core" sequences. The 16-bit sequences as provided by the standard are chosen to have good correlation properties. In other words, their autocorrelations are dirac-like and their mutual cross-correlations are small, regardless of offset. To each 16-bit "core" sequence, five (5) leading bits (which are identical to the last five (5) of the sixteen (16) bits) and five (5) trailing bits (which are identical to the last five (5) of the sixteen (16) bits) are added to form a 26-bit sequence. This improves the likelihood of making a quick

channel estimate for situations in which the training sequence position is known to within five (5) symbol times (e.g., correlation with the original 16-bit sequence is bias-free).

5 Consequently, with respect to the calculation of the correlates with the full 26-bit sequences, it is noted that the last ten (10) bits of each training sequence are identical to the first ten (10) bits: $c_i = c_{i+16}$. This can be verified by comparing values in the table 320 (of FIGURE 10 3B). Thus, $d_i = d_{i+16}$. Equation (1), with N values being equal to the exemplary 26 values, can therefore now be manipulated to:

$$\begin{aligned}
 C_j &= 2 \sum_{i=1}^{10} (d_i \cdot (a_{i+j} + a_{i+j+16})) + 2 \sum_{i=11}^{15} (d_i \cdot a_{i+j}) - S_j = \\
 &= 2 \sum_{i=1}^{10} (d_i \cdot b_{i+j}) + 2 \sum_{i=11}^{15} (d_i \cdot a_{i+j}) - S_j
 \end{aligned}
 \tag{5}$$

In Equation (5), $b_k = a_k + a_{k+16}$ is a *second-order term*.

Referring now to FIGURE 3E, an exemplary calculation of a correlate in graphical form in accordance with the third embodiment of the present invention is illustrated generally at 380. In the graphical calculation 380, the second-order

5 terms b_k are introduced. Because the leading and trailing bits (as marked with dashed lines) of the sequence d are identical, each second-order term is given by the expression: $b_k = a_k + a_{k+16}$. As observable from the table 320 (of FIGURE 3B), both possible values (e.g., "1" and "-1") occur for each

10 of the values of the index i (with the exception of $i = 16$). Consequently, each of the a and b terms in Equation (5) (with the exception of the first order term a_{16}) contributes to some of the respective eight (8) correlates, but not to all of them. As a result, all of the second-order terms should

15 be updated for every new offset j . The corresponding computational effort equates to ten (10) additions.

Of these ten (10) additions, nine (9) may be avoided by implementing the following method: The index expression of the b term is symmetric in i and j . Consequently, when j is

incremented, only one new b term need be calculated. The other terms are already available from the previous value of j , and this previous value of j may be reused:

$$b_{j+1} \leftarrow b_{j+2}$$

...

$$b_{j+9} \leftarrow b_{j+10}$$

$$b_{j+10} \leftarrow a_{j+10} + a_{j+26}.$$

Multiple values of the received sequence that correspond to single, identical (due to repetition) values in the training sequence may be added together prior to multiplication, thus saving multiplications equal to, for example, the number of repeated values. In Equation (5), $b_k = a_k + a_{k+16}$ is a second-order term. Notably, the index expression of the b term is symmetric in i and j . As a result, when j is incremented, advantageously only one new b term need be calculated. The other terms are already available from the previous value of j and may therefore be "reused". (It should also be noted that another

multiplication by "2" has been introduced, but multiplication by "2" is a cheap operation.)

Referring now to FIGURE 4C, an exemplary correlation equation manipulation in accordance with the third embodiment of the present invention is illustrated at block 455C. As indicated by block 455C, the above described manipulation of the correlation equation modifies the correlate equation(s) so that the number of products to be calculated is less than the number of values in a training sequence. Specifically, in this GSM example, because ten (10) values (e.g., a ten-value subsequence, two (2) five-value subsequences, etc.) are repeated within the 26-bit training sequence according to the GSM standard, only sixteen (16) products need to be calculated (e.g., 26 original bits minus 10 repeated bits results in 16 remaining, non-repeated bits to be multiplied). It should be understood that the real value to the third embodiment, when used in conjunction with the second embodiment, derives from a reduction in the number of additions because no product is actually calculated in practice (since d_i is either one (1) or zero (0)).

With respect to a fourth embodiment in accordance with the present invention, it is reemphasized that each of the a and b terms in Equation (5) (with the exception of a_{16}) contributes to some of the eight (8) correlates, but not to all of them. (It is noted that the j term is dropped from the index expressions hereinbelow for the sake of simplicity.) With reference now to FIGURE 5, a table of an exemplary correspondence between first and second-order terms and correlates in accordance with the present invention is illustrated at table 500. Correlate equation terms 505 are composed of first-order terms 505a and second-order terms 505b. The table 500 also lists the indices of the a terms 510 from one (1) to twenty-six (26). For example, the b_4 second-order term is noted to be the sum of the a_4 and the a_{20} terms because the second-order b terms correspond to pairs of first-order a terms in accordance with Equation (5). Furthermore, the table 500 includes 8-bit words 515 that indicate to which of the eight (8) correlates each a (and b)

term contributes. Thus, the term a_{11} contributes to the third, fourth, fifth, and sixth correlates.

It is noted that common subexpression elimination may be applied when referring to the table 500 to minimize the number of mathematical operations needed to produce the eight (8) correlates from the first-order and second-order terms. It is beneficial to "reuse" as many of the partial results as possible in the calculations of the different correlates. An exemplary result of such "reuse" is shown below (where t represents "third-order (intermediate) terms" and r represents the results):

$$t_1 = a_{12} + b_2 + b_{19}$$

$$t_2 = b_3 + b_6 + b_8$$

$$t_3 = b_9 + b_{10} + t_2$$

$$15 \quad t_4 = a_{12} + b_1 + b_7$$

$$t_5 = a_{11} + b_7 + b_9$$

$$t_6 = b_6 + t_1$$

$$t_7 = a_{15} + b_5$$

$$t_8 = a_{13} + t_3$$

$$20 \quad r_1 = a_{15} + t_3$$

$$r_2 = a_{12} + a_{14} + t_7 + t_8$$

$$r_3 = a_{13} + a_{15} + b_8 + t_1$$

$$r_4 = a_{14} + b_8 + t_6$$

$$r_5 = a_{14} + b_4 + b_5 + b_{10} + t_5$$

$$r_6 = b_5 + t_6$$

$$r_7 = t_4 + t_8$$

5 $r_8 = b_2 + t_2 + t_4 + t_7$

10 The total number of additions above is thirty-one (31).
The correlates may then be formed by subtracting S_j from
double the r values. In total, each new sample (i.e., each
new offset value of j in Equation (1)) requires 42
add/subtract operations in an exemplary GSM embodiment.
Specifically, one (1) operation is required to produce the
new second-order b term, thirty-one (31) are required to
produce the eight (8) separate r values, two (2) are
necessary for updating S_j , and eight (8) operations are
15 needed to subtract S_j from $2 \cdot r$, which ultimately produces the
correlates. Referring now to FIGURE 4D, an exemplary
correlation equation manipulation in accordance with the
fourth embodiment of the present invention is illustrated at
block 455D. As indicated by block 455D, the above described

manipulation of the correlation equation modifies the correlate equation(s) by eliminating common subexpressions to thereby "reuse" intermediate values in the determination of the final results.

5 With respect to a fifth embodiment in accordance with the present invention, it is noted that the values of n_d for the GSM training sequences of the table 320 range from eleven (11) to sixteen (16). When $n_d > 13$, fewer first-order terms are required for computing $-C_j$ than for computing C_j . The
10 sign of a correlate matters only if the absolute value of the correlate is sufficiently large. If the absolute vale of the correlate is not sufficiently large, then no interfering signal corresponding to that training sequence is present, and the value may be disregarded. In accordance with this
15 fifth embodiment, $-C_j$ is computed when it requires fewer operations and then the sign is changed only when the correlate is to be subjected to further processing as an interferer (which is rarely in most real-world applications).

20

20

$$t_4 = a_{14} + b_5 + t_3$$

$$t_5 = a_{15} + b_8$$

$$t_6 = b_3 + b_6 + t_5$$

$$t_7 = b_1 + t_1$$

5 $t_8 = b_6 + t_2$

$$t_9 = b_9 + b_{10}$$

$$r_1 = a_{12} + b_5 + t_6 + t_7$$

$$\bar{r}_2 = a_{15} + a_{16} + b_2 + t_4$$

$$r_3 = b_5 + t_8$$

10 $r_4 = b_7 + t_4 + t_9$

$$r_5 = a_{14} + b_8 + t_8$$

$$r_6 = a_{13} + t_2 + t_5$$

$$\bar{r}_7 = a_{16} + t_3 + t_7$$

$$r_8 = t_6 + t_9$$

15 Referring now to FIGURE 4E, an exemplary correlation equation manipulation in accordance with the fifth embodiment of the present invention is illustrated at block 455E (see also related block 465E). As indicated by block 455E, the above described manipulation of the correlation equation (or
20 the calculation of the result(s) thereof) modifies the correlate equation(s) such that a negative of a correlate result is determined if a number of non-zero first-order

terms is greater than a (predetermined) threshold. This implies that the number of operations to calculate the overall set of correlates for all training sequences is minimized to the extent possible by selectively calculating
5 C_j or $-C_j$ for each given training sequence. And as indicated by block 465E, the actual correlation result, for those training sequences for which the inverted correlate was calculated, is determined only if the absolute value of the inverted correlate is greater than a (predetermined)
10 threshold.

With respect to a sixth embodiment in accordance with the present invention, it is noted that any two or more of the first through the fifth embodiments, as described herein above, may be advantageously combined to synergistically
15 further reduce the number of mathematical operations required to produce the correlates. As noted above, implementing all of the first through the fifth embodiments in a GSM wireless network can reduce the number of mathematical operations to one-fifth ($1/5$) of the number required in a straightforward
20 correlation calculation.

The embodiments described above may be applied in various situations. For example, the principles of the present invention may be applied to calculations of multiple correlation values in wireless systems other than GSM.

5 However, it should be understood that certain embodiments are particularly advantageous in particular situations, depending, for example, on the particular properties of (i) the training sequences and (ii) the overall training sequence set. As a first example, the first and second embodiments
10 are particularly advantageous when the training sequences are composed of two distinct values (e.g., "+1" and "-1"). Situations with two distinct values are the most-common cases. Extensions of the first and second embodiments to cases with more than two values are possible, but the benefit
15 is typically less. As a second example, because the third embodiment is at least partially based on how the current GSM training sequences are constructed, the third embodiment is particularly applicable when the sequences (e.g., training sequences) contain repeated subsequences.

As a third example and with respect to the fourth embodiment, it should be noted that a small change in the subexpression tree may produce an identical result at very similar cost. And as a fourth example, the fifth embodiment
5 is likely to be particularly helpful when the values of n_d span a wide range. However, the fifth embodiment is not guaranteed to be helpful because the influence of the common subexpression elimination (e.g., of the fourth embodiment) is unpredictable, but the probability of finding a
10 combination that features a greater number of common subexpressions grows as 2^N , when N is the number of correlates. For GSM where $N = 8$, among the resulting two hundred and fifty-six (256) combinations, several exist that reduce the overall operation count by two (2) (e.g., by
15 approximately five percent (~5%)).

It should also be noted that particular embodiments are particularly beneficial in certain implementations (e.g., DSP, ASIC, etc.). For example, the third embodiment benefits from modulo addressing of a circular buffer, but it is
20 therefore economically less useful if the cost of memory is

larger than the cost of an addition. Also, if operations are performed on a DSP, the executable code for the transformed algorithm may be larger than the original code (e.g., because the transformations reduce the regularity of the initial description). Other constraints and/or factors to consider will become apparent to one skilled in the art after reading and understanding the principles of the present invention.

It should further be noted that common subexpression elimination as described hereinabove with respect to the fourth embodiment focuses on minimizing the number of add/subtract operations. Other optimization criteria may also be considered, depending on what is appropriate for the implementation technology. An important property to consider in the context of realizing the fourth embodiment in a particular environment is that some intermediate results produced in the computation of one correlate are advantageously re-used to form other correlates.

Although preferred embodiment(s) of the method, system, and apparatus of the present invention have been illustrated in the accompanying Drawings and described in the foregoing

Figure 6 shows the effect of the concentration of the monomer on the polymerization rate. The rate increases with increasing monomer concentration, and the reaction order with respect to monomer concentration was determined to be 0.78.

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